Exercises : [from the slides] –

[!] Most of them have already been treated in the exercise documents or in the Recap-kinematics.

① 2D-problem – ⚠

Exercise:

1a) **R**(ϑ= 0) =?

1b) **R**(-ϑ) =?

1c) **R** (ϑ)$^{-1}$=? **R**^{T} (6)

1d) **v**3 = **R**(θ2)**v**2 = **R**(θ2)**R**(θ1)**v**1 = **R**(?)**V**1 ? (7)

1e) **R**(θ2)**R**(θ1) = **R**(θ1)**R**(θ2)?

if we consider $R_z(\theta)$ – ⊙s in 2D

1-a

$$R_z(\theta=0) = \begin{vmatrix} \cos(\theta=0) & -\sin(\theta=0) \\ \sin(\theta=0) & \cos(\theta=0) \end{vmatrix} = \boxed{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}$$

Observ

In all cases, the rotation matrix around the origin equals to the Matrix Identity ☒

1-b-c

$$R(-\theta) = \begin{vmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{vmatrix} = \begin{vmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{vmatrix}$$

$$R^T(\theta) = \begin{vmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta \end{vmatrix}$$

Observ

In all cases $R(-\theta) = R^T(\theta) = R^{-1}(\theta)$ ☒

1-d

$$R(\theta_2)R(\theta_1) = \begin{vmatrix} c_2 & -s_2 \\ s_2 & c_2 \end{vmatrix}\begin{vmatrix} c_1 & -s_1 \\ s_1 & c_1 \end{vmatrix}$$

$$= \begin{vmatrix} c_1 c_2 - s_1 s_1 & -(c_2 s_1 + c_1 s_2) \\ s_2 c_1 + c_2 s_1 & -s_1 s_2 + c_1 c_2 \end{vmatrix}$$

$$= \begin{vmatrix} c_{12} & -s_{12} \\ s_{12} & c_{12} \end{vmatrix}$$

$$R(\theta_1)R(\theta_2) = \begin{vmatrix} c_{12} & -s_{12} \\ s_{12} & c_{12} \end{vmatrix}$$ same as before

That makes sense: successive rotations around the same axis of rotation is equivalent to a single rotation of a sum of the 2 rotations

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Example of use: Let us assume a movement of the point $\underline{v}1$ in two phases composed of

1.) rotation (always around the origin O for all this section 2.1.2) by an angle $\theta 1$

2.) translation of $\underline{t}1$

to get $\mathbf{v}2 = \mathbf{R}(\theta 1)\, \mathbf{v}1 + \mathbf{t}1$

Let us apply again a motion composed of two phases ($\mathbf{R}(\theta 2)$, $\mathbf{t}2$) to $\mathbf{v}2$:

3.) rotation around the origin O by an angle $\theta 2$

4.) translation of $\mathbf{t}2$

to finally get $\mathbf{v}3$:

$$\mathbf{v}3 = \mathbf{R}(\theta 2)\, \mathbf{v}2 + \mathbf{t}2 = \mathbf{R}(\theta 2)\, \mathbf{R}(\theta 1)\, \mathbf{v}1 + \mathbf{R}(\theta 2)\, \mathbf{t}1 + \mathbf{t}2 \quad (9)$$

Steps 1–2 $\to$ Homogeneous transformation H_1

Steps 3–4 $\to$ Homogeneous transformation H_2

These four consecutive movements ($\mathbf{R}(\theta 1)$, $\mathbf{t}1$, $\mathbf{R}(\theta 2)$, $\mathbf{t}2$) can be expressed in a single homogeneous matrix which is calculated simply by matrix products.

Exercise: Perform this operation with the help of two homogeneous matrices. Calculate the homogeneous matrix of the complete operation by matrix product. This result will contain the total translation and the total rotation equivalent to the four movements.

$$\vec{v}_3 = H_2 \cdot \vec{v}_2 = H_2\, H_1\, \vec{v}_1$$

$$\vec{v}_3 = \begin{vmatrix} R(\theta_2) & t_2 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} R(\theta_1) & t_1 \\ 0 & 1 \end{vmatrix} \vec{v}_1$$

$$\vec{v}_3 = \begin{vmatrix} R_2 R_1 & R_2 t_1 + t_2 \\ 0 & 1 \end{vmatrix} \vec{v}_1$$

equivalent combined rotation

equivalent translation

③

Exercise 2 Homogeneous matrices -

Ex 2a): Pure rotation? Pure translation?

Ex. 2b) Translation of t, then rotation of ϑ

Ex. 2c) Rotation, then translation

Reverse? (Check it !)

2.a

$$H_R = \begin{vmatrix} R & 0 \\ 0 & 1 \end{vmatrix}$$ - pure rotation, no translation

$$H_T = \begin{vmatrix} I & t \\ 0 & 1 \end{vmatrix}$$ - pure translation, No rotation ($R(\theta=0) = I$)

2.b

$$\xrightarrow[H_T]{t} \xrightarrow[H_R]{\theta}$$

$$H = H_R \cdot H_T = \begin{vmatrix} R & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} I & t \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} R & Rt \\ 0 & 1 \end{vmatrix}$$

$$\xrightarrow[H_\theta]{\theta} \xrightarrow[H_R]{t}$$

$$H = H_T \cdot H_R = \begin{vmatrix} I & t \\ 0 & 1 \end{vmatrix} \begin{vmatrix} R & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} R & t \\ 0 & 1 \end{vmatrix}$$

observe

Non commutative transformation

4

Exercise (on slides):

a.) $K_{DGM}(\vartheta i)$ = (rot ϑ_1 around $[0, 0]^T$) then (rot ϑ_2 around $\underbrace{[L_1c_1, L_1s_1]^T}_{P_1}$) then (rot $\vartheta 4$ around $\underbrace{[L_1c_1 + L_2c_{12},]^T}_{P_2}$)

b.) $K_{DGM}(\vartheta i)$ = (rot ϑ_4 around $[\ ?]^T$) then (rot ϑ_2 around $[\ ?]^T$) then (rot ϑ_1 around $[\ ?]^T$)

c.) Calculate $K_{DGM}(\vartheta i)$ of the SCARA. Which method will be simpler? Where does the rotation matrix come from around ϑ_1, left or right in the chain of multiplications?

Exercise (on slides): A working point of the wrist is at a horizontal distance $L4$ from the axis of $\vartheta 4$ with $P(\vartheta i = 0) = [L1 + L2 + L4, 0, 0, 1]^T$. Give the position of this point as a function of the robot variables $P(\vartheta i)$.

a- Rot θ_1, around $(0, 0)$ $\rightarrow$ $H_1 = \begin{vmatrix} R & 0 \\ 0 & 1 \end{vmatrix}$

Rot θ_2, around $P_1 = (L_1C_1, L_1S_1)^T \rightarrow H_2 = \begin{vmatrix} R_2 & P_1 - R_2 P_1 \\ 0 & 1 \end{vmatrix}$

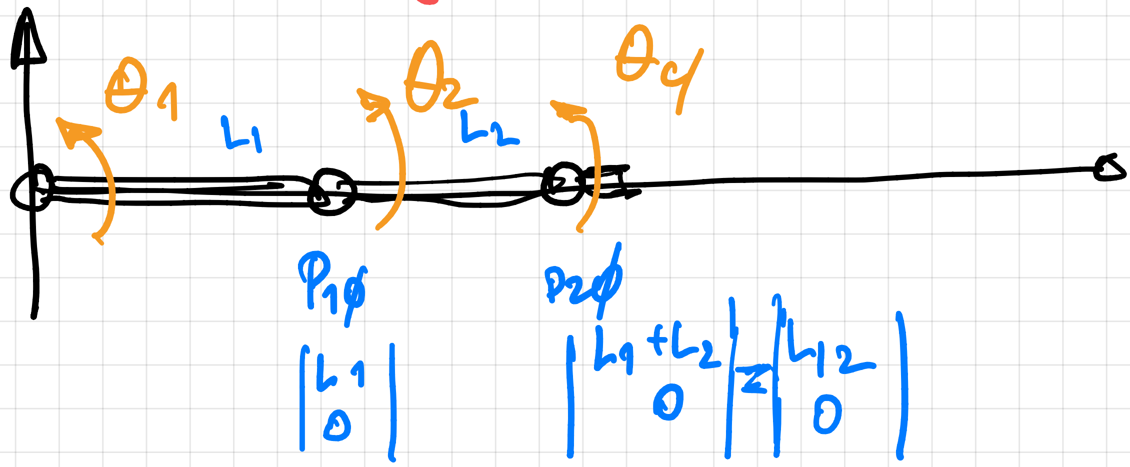
Rot θ_4, around $P_2 = (L_1C_1 + L_2C_{12}, L_1S_1 + L_2C_{12})$

$$H_4 = \begin{vmatrix} R_4 & P_2 - R_4 P_2 \\ 0 & 1 \end{vmatrix}$$

$$H = H_4 \cdot H_2 \cdot H_1 = K_{DGM}(\theta_1, \theta_2, \theta_4)$$

to develop

b- We can perform the rotation at the inverse starting from θ_4.



Rot θ_4, around $P_{20} = (L_{12}, 0) \rightarrow H_4 = \begin{vmatrix} R_4 & P_2 - R_4 P_2 \\ 0 & 1 \end{vmatrix}$

Rot θ_2, around $P_{10} = (L_1, 0) \rightarrow H_2 = \begin{vmatrix} R_2 & P_{10} - R_2 P_{10} \\ 0 & 1 \end{vmatrix}$

Rot θ_1, around origin $\rightarrow H_1 = \begin{vmatrix} R_1 & 0 \\ 0 & 1 \end{vmatrix}$

$$H^* = H_1 \cdot H_2 \cdot H_4$$

/ to_develop

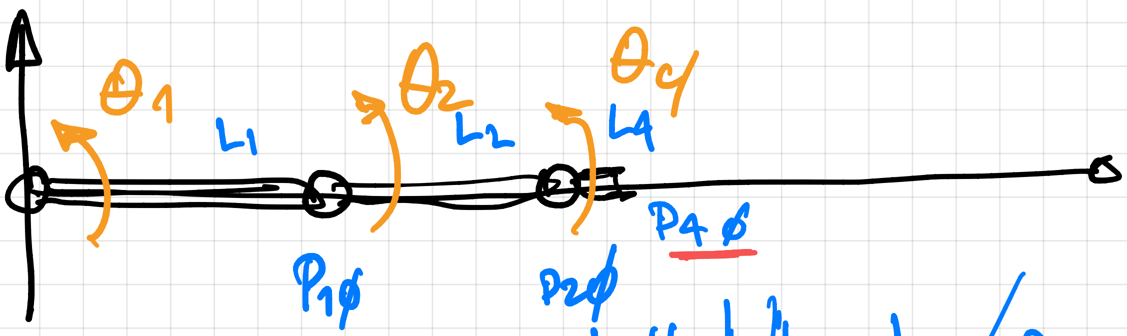
c- the second method, is simpler

It is the one presented in the slide " Methodology to establish DGM in 4 steps"

$$K_{DGM}(\theta) = H^* \, P_{20} = \begin{vmatrix} L_{12} \\ 0 \end{vmatrix}$$

H^* : answer b

D – extremity of the wrist @ distance L_4 from $P_{2\emptyset}$



$P_{1\emptyset} = \begin{vmatrix} L_1 \\ 0 \end{vmatrix}$ $\quad P_{2\emptyset} = \begin{vmatrix} L_1 + L_2 \\ 0 \end{vmatrix} = \begin{vmatrix} L_{12} \\ 0 \end{vmatrix}$ / $P_{40} = \begin{vmatrix} L_{124} \\ 0 \end{vmatrix}$

$$K_{DGM} = H^* \, P_{40} = H^* \begin{vmatrix} L_{124} \\ 0 \end{vmatrix}$$

to develop | detailed answer is in the series.

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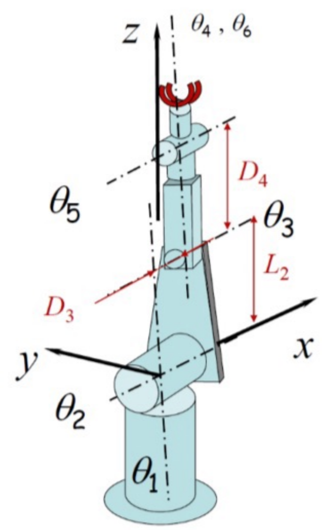
Exercise 3

The homogeneous matrices $\mathbf{K}_5$ and $\mathbf{K}_6$ of the DGM of the PUMA (see figure below) are given in the course (see lecture slides). Give the missing matrices $\mathbf{K}_i$, for i = 1,2,3,4.



The homogeneous matrices of the PUMA DGM (according to the generalized coordinate system of the course) are as follows:

$$K_4 = \begin{pmatrix} c_4 & -s_4 & 0 & D_3 v_4 \\ s_4 & c_4 & 0 & -D_3 s_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & L_2 s_3 \\ 0 & s_3 & c_3 & L_2 v_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$K_1 = \begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

6 [!] Not mandatory for you ☺

Exercise : Find the homogeneous transformation matrix leading from the form {A, B, C} to {A ', B', C '}.

$A = [1\ 0\ 0]^T$, $B = [0\ 0\ 0]^T$ and $C = [0\ 1\ 0]^T$ to $A' = [1\ 0\ 1]^T$, $B' = [1\ {-1}\ 1]^T$ and $C' = [0\ {-1}\ 1]^T$

What are the axis, the angle of rotation, the translation in the direction of the axis? The solution is easily found with a small drawing. Imagine the three points linked to a solid.

Exercise:

4a) Is the previous paragraph completely correct?

4b) Find the center of rotation 1.) by a drawing, 2.) using the previous formula 3.) by looking for an eigenvector of the homogeneous matrix.

4c) Find the homogeneous matrix which describes a rotation of 60 ° around O

4d) Find the homogeneous matrix which describes a translation of un indirection x , then a rotation of 60 ° around O

4e) Find the homogeneous matrix which describes a 60 ° rotation around [1,1] T.

4.f) An object with two points **v**1, **w**1 is moved so that these points are found at locations **v**2, **w**2 .

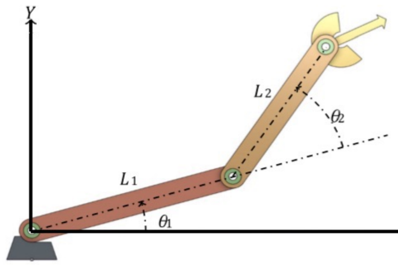
$\mathbf{v}1 = [1.0]^{T,}$ $\mathbf{w}1 = [1.1]^{T,}$ $\mathbf{v}2 = 0.5\ [1 - \sqrt{3}.1 - \sqrt{3}]^{T,}$ $\mathbf{w}2 = 0.5\ [2 - \sqrt{3}.1]^T$

Find the homogeneous matrix, θ, $\boldsymbol{p}$ (graphical solution) which describes this displacement.

Exercise 4

Find the IGM (Inverse geometric model) of a 2 DOF planar robot (see figure below): given x and y, what are θ_1 and θ_2?

$$x = L_1 c_1 + L_2 c_{1+2}$$
$$y = L_1 s_1 + L_2 s_{1+2}$$



→ Solve a quadratic equation.

Hint: use the trigonometric formulas for the sine and cosine of the sum of two angles, as well as the one of the sum of squares of sine and cosine, to obtain a system of 4 equations with 4 unknowns, and discuss the results.

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